

Correlograms: reading a series' memory

How much does this month resemble last month, or the same month last year? A correlogram answers for every lag at once. It is how you tell whether a series has a trend or a season to remove, which time-series model fits it, and, after fitting, whether the model left anything behind.

Function	<code>fx.RiskACF</code> , with <code>fx.RiskDifference</code> and <code>fx.RiskSARIMAResidual</code> in the example
In XL Edge	XL Edge tab > Productivity Tools > Stat Tools > Insert Time Series > Test a series > Correlogram (ACF, PACF, Ljung-Box)
Example	<i>XL Edge Example - Correlogram.xlsx</i> (same folder). Every number in this guide is read from it.
Method	Sample autocorrelations (divisor n); Bartlett's bands; Durbin-Levinson partial autocorrelations; the Ljung-Box Q test.

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1 The idea: ACF, PACF and the test

The **autocorrelation at lag k** is the correlation between a series and itself shifted k periods: lag 1 compares each month with the month before, lag 12 with the same month a year earlier. Plotted for lags 1, 2, 3 ... it is the **ACF**, the series' memory profile.

The ACF mixes direct and indirect links: if this month echoes last month, and last month echoed the month before, lag 2 looks correlated even with no direct link. The **partial autocorrelation (PACF)** at lag k removes that echo: it is the correlation at lag k after allowing for lags 1 to k - 1. Together the two identify ARMA models (section 8).

Each bar comes with a **band**. A bar outside it is statistically significant at the chosen level (5% by default). The **Ljung-Box test** pools all the bars up to a lag into one question: is the series *white noise*, with no autocorrelation at all?

2 Using the ribbon

XL Edge tab > Productivity Tools > Stat Tools (arrow) > Insert Time Series > Correlogram (ACF, PACF, Ljung-Box)

It sits in the *Test a series* section of the Time Series menu.

- 1 **Select the series:** one column, oldest first (a header above it names the block), or any cell of a spilled result.
- 2 **Highest lag:** `auto` uses $\text{MIN}(10 \log_{10} n, n - 1)$, which is 20 for 108 months; type 24 to see two full years of a monthly series.
- 3 **Significance level** for the bands and the test: 0.05.
- 4 **ModelDF:** 0 for a raw series; for the residuals of a fitted model, the number of ARMA coefficients it estimated (section 6).
- 5 **Pick an empty cell.** The block is (Lags + 3) rows by 10 columns.

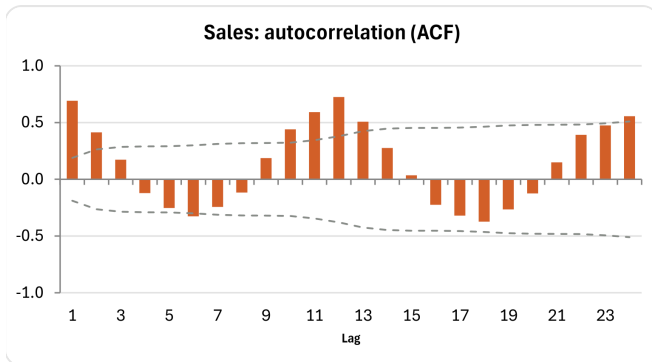
3 Reading the block

```
series, highest lag, alpha (0.05), ModelDF (0), name for the title
=fx.RiskACFA(Data!C5:C112, 24, , , "Sales")
```

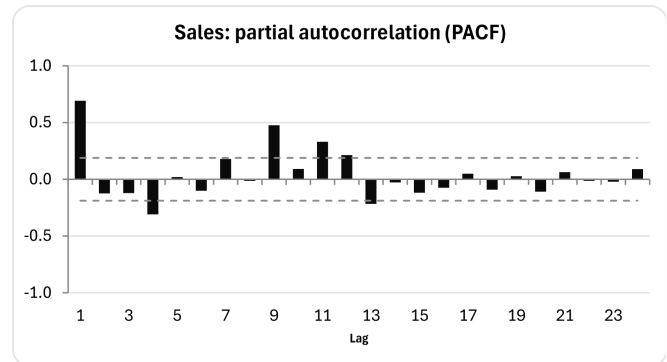
Column	Meaning
Lag	0 (always 1) to the highest lag.
ACF, Lower, Upper	The autocorrelation and its band. Bartlett's band <i>widens</i> with the lag, because each bar's uncertainty includes the bars before it.
PACF, Lower, Upper	The partial autocorrelation and its band, $\pm z/\sqrt{n}$, the same at every lag.
Q-stat, p-value	Ljung-Box statistic for lags 1 to this one, and its p-value.
White noise?	"Yes" when $p \geq \alpha$: no autocorrelation detected up to this lag.

The example uses the 108 months of sales from *SARIMA - Example Data.xlsx*. Its charts are ordinary Excel charts drawn from the block's columns: bars for the ACF or PACF, dashed lines for the band.

4 Step 1: the raw series



Sales, ACF (sheet 1 Sales).



Sales, PACF.

Reading it. Lag 1 is 0.69: a strong month tends to follow a strong month. The bars fall to -0.33 at lag 6 (opposite seasons) and climb back to 0.73 at lag 12 and 0.56 at lag 24. Two messages:

- **A season.** Peaks every 12 lags mean each month resembles the same month in other years.
- **Not stationary.** Large autocorrelations that persist across years mean the level and the pattern carry forward; the series has no fixed average to return to.

The Ljung-Box test agrees emphatically ($Q = 432.8$ over 24 lags, $p < 0.001$). Note the band: ± 0.19 at lag 1 but ± 0.38 at lag 12, widened by all the large bars before it. A raw correlogram like this one is not for picking a model; it says "remove the trend and season first".

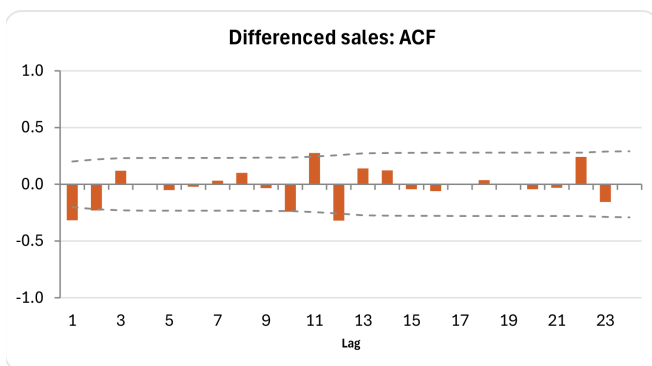
5 Step 2: difference, then identify

Differencing removes trend and season: take each month minus the month before (lag 1), then each result minus the same month a year before (lag 12). `fx.RiskDifferenceλ` does both; 95 values remain from 108 (one lost to each difference: $1 + 12$).

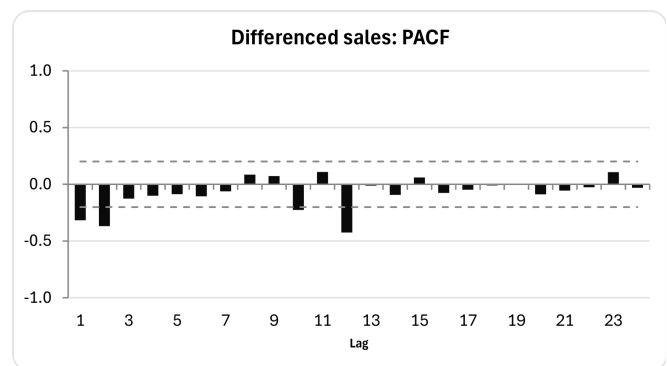
sheet 2 Differenced: difference once at lag 1 and once at lag 12, then the correlogram of the result

N6: `=fx.RiskDifferenceλ(Data!C5:C112, 1, 1, 12)`

A6: `=fx.RiskACFλ(N6#, 24, , , "Differenced sales")`



Differenced, ACF: negative spikes at 1 and 12.



Differenced, PACF: decaying from lag 1, again near 12.

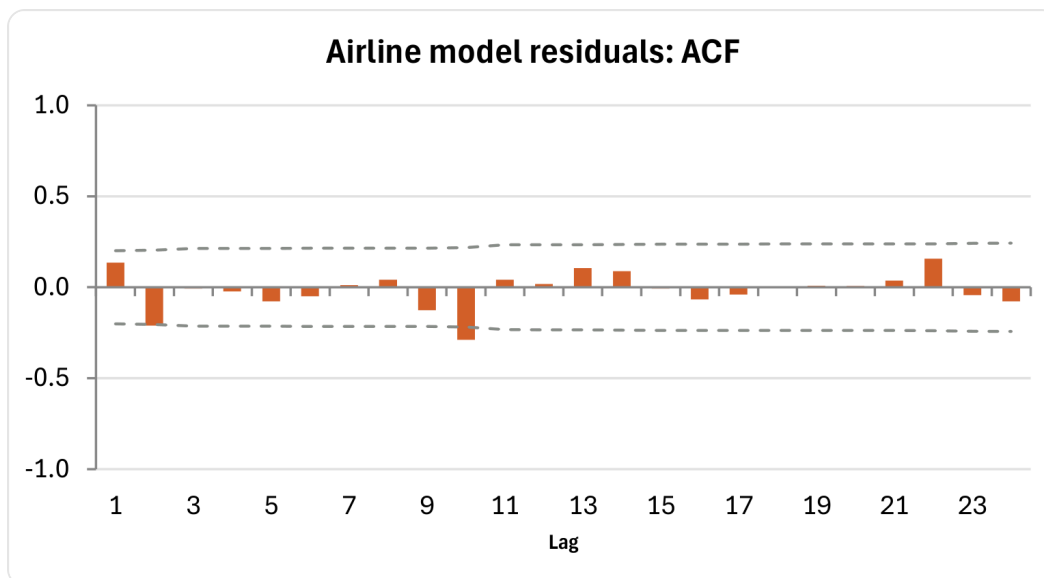
Reading it. The memory is gone except at two places: lag 1 (-0.32 , band ± 0.20) and lag 12 (-0.32 , band ± 0.26). Both are negative, and in between the bars are small. That shape, an ACF that *cuts off* after a lag while the PACF *tails off* ($-0.32, -0.37 \dots$), is the signature of moving-average terms: one at lag 1 and one at the seasonal lag 12. Together with the two differences, that is SARIMA (0,1,1)(0,1,1)₁₂, the classic **airline model**.

The model even predicts the small bars at lags 11 (0.28) and 13 (0.14): multiplying a lag-1 and a lag-12 term creates echoes at 11 and 13 with the opposite sign to the spikes. Lags 2 (-0.23 , band ± 0.22) and 10 (-0.24 , band ± 0.24) only just cross; a marginal bar or two among 24 is expected by chance (section 7), and the residual check below is the real test.

6 Step 3: check the residuals

A good model leaves residuals (one-month forecast errors) that are white noise: nothing predictable left. The example fits the airline model with `fx.RiskSARIMAFitA` (MA1 = -0.642 , SMA1 = -0.315) and computes its residuals:

```
sheet 3 Residuals: N6 holds the Model block; rows 11-16 of its Estimate column are AR1, AR2,
MA1, MA2, SAR1, SMA1
N6: =fx.RiskSARIMAFitA(Data!C5:C112)
Q6: =fx.RiskSARIMAResidA(fx.RiskDifferenceA(Data!C5:C112, 1, 1, 12),
CHOOSEROWS(CHOOSECOLS(N6#, 2), 11, 12, 13, 14, 15, 16), 12, 0)
A6: =fx.RiskACFA(Q6#, 24, , 2, "Airline residuals")
```



Airline residuals, ACF (sheet 3 Residuals). The spikes at 1 and 12 are gone.

MODELDF: WHY 2

The fitted model has already used two coefficients to remove autocorrelation, so the residuals look whiter than truly random numbers would. Ljung-Box allows for that by subtracting ModelDF from its degrees of freedom: here $24 - 2 = 22$ at lag 24. The test is blank until the lag exceeds ModelDF.

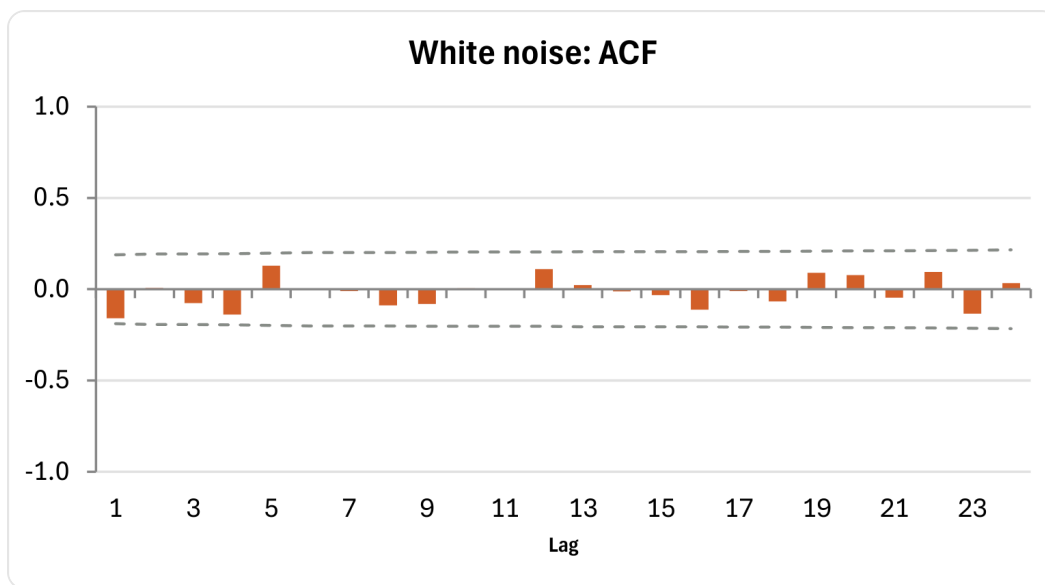
Reading it, candidly. Overall the residuals pass: Ljung-Box over 24 lags gives $p = 0.277$ ("Yes"), and the lag-12 bar has dropped to 0.02. But it is not a perfectly clean bill: still, 2 bars cross the band, at lag 2 (-0.21 , band ± 0.20) and lag 10 (-0.29 , band ± 0.22). The test over the first 3 lags rejects white noise ($p = 0.013$). The lag-2 bar hints at a second moving-average term, $(0,1,2)(0,1,1)_{12}$. The *SARIMA* guide backtests that model against the airline model: it does a little better, mostly over a 24-month holdout. The hint was real, but small.

7 What chance alone looks like

Sheet 4 *White noise* runs the same test on 108 independent random numbers, a series with nothing in it. In this draw the count of ACF bars outside the band is 0 of 24, and Ljung-Box gives $p = 0.738$. With a 95% band you should expect about one bar in twenty to poke out by chance (1.2 of 24), even in pure noise.

Two consequences:

- **Don't chase single marginal bars**, especially at lags with no meaning for the business (lag 7 in monthly data).
- **Do take seriously** bars at meaningful lags (1, 2, the seasonal lag) and anything the Ljung-Box test confirms.



White noise, ACF. This is what "nothing left" looks like.

8 A pattern guide

What you see	What it suggests
ACF starts near 1 and decays very slowly	Trend or random walk: difference ($d = 1$). Check with Unit Root Tests.
Peaks at the seasonal lag and its multiples (12, 24 ...)	Seasonality: seasonal difference ($D = 1$) or seasonal terms.
ACF cuts off after lag q , PACF tails off	MA(q): q moving-average terms.
PACF cuts off after lag p , ACF tails off	AR(p): p autoregressive terms (Fit ARMA, or Yule-Walker / Burg).
Both tail off	ARMA(p, q): try small p and q and compare.
A single spike at the seasonal lag (ACF)	Seasonal MA: SMA1 ($Q = 1$). With a lag-1 spike too, expect echoes at $s - 1$ and $s + 1$.
A large negative lag-1 ACF (below -0.5) after differencing	Over-differenced: try one difference fewer.
All bars inside, Ljung-Box $p \geq \alpha$	White noise: nothing left to model.

9 Practical advice

- **Use enough data.** At least 50 observations, and four or five full seasons for a seasonal series. The bands widen as n falls.
- **Read the ACF and PACF together,** and after any differencing, never only on the raw series.
- **One-off events distort everything.** A strike, a pandemic month or a data error creates spurious bars. Look at a line chart of the series first.
- **The correlogram suggests, the backtest decides.** Use it to shortlist models, then compare them out of sample (SARIMA guide, section 8).
- **Other residual checks.** The same function works on regression residuals (Regression Residuals block) or VAR residuals, one column at a time. For regression, also see *Serial Correlation (Breusch-Godfrey)*.

10 Argument reference

fx.RiskACFλ(X, [Lags], [Alpha], [ModelDF], [Name])

Argument	Default	Meaning
X	required	The series, oldest first, one column.
Lags	MIN(INT(10 log ₁₀ n), n – 1)	Highest lag shown.
Alpha	0.05	Significance level of the bands and the Ljung-Box test.
ModelDF	0	Coefficients already fitted (p + q of an ARMA, plus seasonal ones); Ljung-Box uses Lag – ModelDF degrees of freedom.
Name	"Series"	Name for the title row.

Output, (Lags + 3) × 10: a title; Lag, ACF, Lower, Upper, PACF, Lower, Upper, Q-stat, p-value, White noise?. ACF bands: $\pm z \sqrt{((1 + 2 \sum \text{earlier ACF}^2)/n)}$ (Bartlett); PACF bands: $\pm z/\sqrt{n}$.

fx.RiskDifferenceλ(X, Diff, SeasonalDiff, Season)

Argument	Default	Meaning
Diff, SeasonalDiff	required	0 or 1 each: difference at lag 1, then at lag Season. The result is shorter by Diff + Season × SeasonalDiff.

References: Bartlett (1946), *JRSS Supplement* 8(1); Durbin (1960), *Review of the ISI* 28(3); Ljung and Box (1978), *Biometrika* 65(2); Box, Jenkins and Reinsel, *Time Series Analysis* (identification, the airline model). XL Edge and the fx.Risk... functions:
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